Modeling Spatial Activity Distributions in Complex Urban Conditions: The Markov Chain Model for Weighting Spaces with Attractors

İPEK ROHLOFF
Mount Holyoke College

KURT ROHLOFF
Raytheon BBN Technologies

INTRODUCTION

Cities are becoming more complex due to recent developments such as rapid urbanization and green open spaces implemented as urbanism solutions. Understanding the complexity based on these new developments calls for innovative spatial analysis techniques and digital representation methods. A number of innovative analysis techniques and representations have been offered within the space syntax methodology. Space syntax developed by Bill Hillier and Julianne Hanson in 1984,[1] is considered a robust theory portraying urban morphologies in terms of human spatial activity distributions. Its quantitative spatial analysis techniques have made an innovative leap within the urbanism theories based on ‘space paradigm,’ which was shaped by a group of urbanists understanding spatial complexity of cities in terms of human spatial experience in spaces between buildings.[2,3,4] With the latest advancements in its methods of modeling spatial activity, space syntax offers a rigorous methodology to forecast potential movement distributions in cities based on street network properties. Street network analysis is based on graph theory and consists of calculating topological relations among spatial units. Computational modeling of street network takes street segments as spatial units and calculates the connectivity among them. Using the connectivity algorithms, the analysis to estimates how likely street segments become destination points (*to-movement*) or thoroughfares (*through-movement*) that attract movement due to their relations with other segments. [5]

Despite its robustness, street network analysis remains limited in its ability to capture potential movement distributions in cities containing grater spatial complexity due to non-organic development patterns. Green open spaces or plazas that have emblematic prominence may also attract movement due to their qualitative content such as effects of greenery on people’s well-being. Exploring the effect of attractors, such as building density and green open spaces, is central to predicting the dynamic and complex spatial conditions in cities and thus being able to understand the implications of dramatic changes in cities. With this in mind, this paper introduces a new approach to analyze potential spatial activity distributions at a finer grain by accounting for complex factors beyond street network properties.

Recent studies investigating more informative and detailed modeling approaches for spatial activity distribution argue that geometric differentiation, building density and other program related properties may impact spatial activity over the effect of street connectivity. We propose a new analysis approach to predict spatial activity distribution as a function of other programmatic and environmental contents of streets in order to augment existing spatial modeling techniques. This new analysis approach, based on Markov chain models,[6] has been applied in several domains with great success.[7,8,9] A specific benefit of the Markov chain approach is that there are mature and widely available computational techniques that support the analysis of these models. (See the Matlab software tool available at www.matlab.com, for example.)

This paper discusses the following questions. (1) How well does the street network analysis capture spatial activity distributions? (2) What other factors may influence movement distribution in cities beyond the effect of network? (3) How can a new analysis technique based on the Markov chain model detect the effect of these factors?

We investigate these questions as follows. In the section immediately following we review previous studies into representations capturing urban complexity and associated spatial activity distributions. In Section 3 we discuss how the latest analysis techniques in space syntax theory account for spatial activity distributions in cities. In Section 4 we discuss two representative cases where street network relations may not be primary determinant of movement. In Section 5 we introduce the Markov chain mathematical model and we discuss its relevance to address limitations of previous approaches in capturing less-tangible and complex predictors of movement. The final section discusses the departure points the Markov chain could offer for understanding complex urban conditions and development patterns.

EXPLORATIONS ON URBAN COMPLEXITY AND ITS REPRESENTATION

Urbanism theories emerging in mid-twentieth century emphasize space and spatial experiences as an alternative to reading urban environment in terms of formal compositions. This emphasis upon space have been a common denominator in the works of geographers such as Edward Soja, David Harvey as well as architects Bernard Tschumi, Rem Koolhas and other urban theorists such as Jan Gehl, William H. White, and Jane Jacobs, who are interested in spatial and programmatic complexity of cities. Bernard Tchumi explored spatial complexity in the disjunctions of space, form and events. While arguing that there is no space without ‘events’ (or programmatic elements), Tschumi still uses formal compositions to express spatial complexity. His Parc de La Villette project expresses spatial complexity through tensions, and conflict between superimposed formal systems (1995). [10] Within such explorations, Bill Hillier’s and Julianne Hanson’s space syntax framework (1984) becomes a departure in understanding spatial complexity of cities without falling into restrictive language of forms, yet in terms of movement that can be generated within the network of spatial components.

With its thrust on graph theory and quantitative expressions of relational aspects of configurations, the space syntax framework has gained a more prominent role as a modeling and forecasting framework that can inform design and planning in addition to being a design style. Despite the highly informative digital representations of quantitative street network analyses, space syntax modeling of cities is far less subjective than the representative mapping and diagramming of people’s spatial experiences promoted by designers like James Corner. [11]

Due to its abstract and quantitative nature, space syntax methodology, and street network analysis in particular have some limitations in predicting complex conditions such as effects of programmatic elements on movement. A number of researchers have discussed these limitations in various contexts and proposed new modeling applications. Raford (2009) discusses the limitations of space syntax are particularly in response to the urban design and planning practices in North America, which do not align with organically growing cities where space syntax theories derived. North American cities have the phenomenon of planning with mobility through highways, which connects settlements in non-spatial ways.[12] Another group of studies addresses limitations associated with the abstract nature of space syntax street network modeling. They develop counter arguments to Hilllier’s (2008) proposition that location of attractors is a function of space network relations. [13] Ratti (2004) argues that factors such as building density and geometry of the urban block are undetected in street network analysis despite the potential effect of these factors on movement.[14] In his later work, Ratti (2005) suggests some applications that can detect the effects of building density and three dimensional sight lines. The effect of other potential attractors such as building density, land-uses and environmental information are addressed more directly by Ståhle (2007, 2008) and Sevstuk (2010) within their proposed analysis applications.[15,16] Ståhle models an analysis application, by combining the space syntax methodology with geographic information systems in order to capture environmental and contextual factors actually shaping people’s understanding of a place. More recently, Sevstuk (2010) develops a comprehensive analysis model analyzing location of retail activity based on an array of variables including visibility, accessibility, density, adjacency and geometry of build environment.

Previous explorations into spatial and programmatic complexity of cities and recent discussions on the effects of programmatic elements confirm the need for improvement in the spatial analysis approaches detecting how attractors, such as building density, natural settings and geometric shape may influence movement. Our argument is that these attractors may make a difference in the way in which street networks are read and prioritized by human cognition. In an effort to develop more informative spatial analysis models, we propose integrating space network analysis with another mathematical model, namely the Markov chain model.

MODELING MOVEMENT ECONOMIES IN CITIES ON THE BASIS OF STREET NETWORK RELATIONS

The theoretical ground of the space network methodology relies on expressing the knowledge of space neither entirely in terms of form nor of human experience, but in terms of interactions between human and built environment.[17] This interaction is understood within formulations of human spatial activity by network relations among spaces. These network relations are modeled as graphs that refer to collections of set of nodes and links. In graph representations of configurations, nodes correspond to spatial units (rooms, street segments) and links represents transitions and connections among those spatial units. The hierarchical relationships among those spatial units are calculated within connectivity algorithms among the nodes, which is independent of geometrical shape and size of the units. These connectivity algorithms fundamentally express the degree to which each spatial unit is connected to neighboring units (local/connectivity), and the entire configuration (global/integration). [18, 19]

Space syntax theory, therefore, explains distributions of human spatial activity on the basis of how well streets are connected to their neighborhood or the entire city. The theory suggests that streets or their segments that are reachable from all other segments by involving the fewest number of other spaces attract movement. Recent advancements in street network modeling propose that street segments that are reached within the least number of turns and minimum sum of angular change are likely to be destination points (Fig.1). This proposition suggests that movement is generated by an economy minimizing the cost of a journey within a street network. The street segment analysis based on least number of turn and minimal angular change showed better correlations with actual human activity in districts of London.[20] Accordingly, this theory and particular propositions of angular segment analysis also explain the concentration of land-uses, in particular commercial activity in certain centers. Land-uses in other words are programmatic elements that are in fact economic entities also migrate those integrated or highly preferable destination points. These programmatic elements, such as shopping centers and other commerce also attract movement. Therefore, movement in cities is distributed within the synergy created between space network and programmatic attractors. The theory suggests movement is mainly determined by the space network which also determines the position of attractors. In other words, “the configuration of the space network is, in and of itself, a primary shaper of the pattern of movement.”[21]

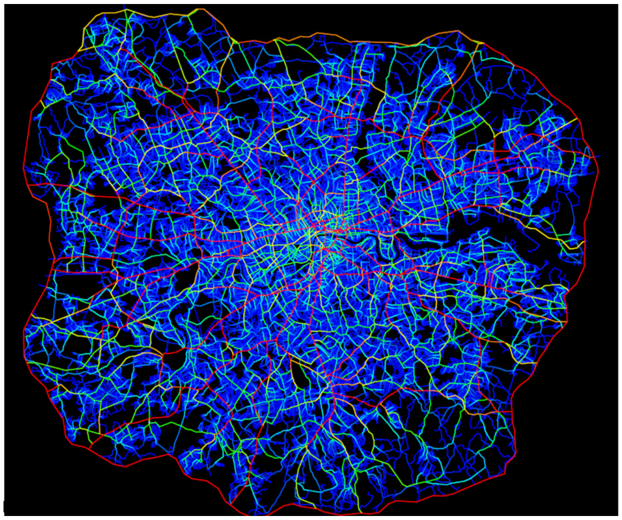

Figure 1. Segment analysis model, showing the main structure of global routes in London. Reddish colors show the segments that are most likely to be thoroughfares. (source: Hillier (2009)).[22] “Spatial Sustainability in Cities: Organic Patterns and Sustainable Forms.” Paper presented at the 7th International Space Syntax Symposium, Stockholm, 2009.)

This proposition disregarding the independent effect of programmatic and other possible attractors on movement is where space syntax is challenged within the academic community. The counter argument is that the prediction of spatial activity merely by street network remains too simplistic, as human spatial activity is actually influenced by a set of variables including perceptions of building size and density, metric distance, geometric shape and environmental content, defined within spatial complexity of cities. Despite acknowledging the partial role of programmatic elements as attractors, space syntax theory accepts street segments as discrete units that have same or similar content within a skeletal system. Moreover, the theory establishes parallels to the way our spatial cognition works in choosing paths within these skeletal system. [23] Within this abstract representation, spaces between buildings such as streets, squares, segments and other open spaces are differentiated only in terms of how easily they are reached within a network. When programmatic and environmental content of these spaces are taken into consideration, space syntax modeling of cities is only a layer of representation, which can be completed with analysis of various other factors such as land-use data, population, building densities, and environmental content.

OTHER FACTORS SUCH AS PROGRAMMATIC ATTRACTORS PREDICTING MOVEMENT DISTRIBUTIONS

These districts may have highly concentrated spatial activity despite their segregated position in the street network. The high concentration of spatial activity in those centers may not be explained within street network modeling unless multiple transportation routes these centers might receive, such as subways and motorways are included in the modeling. However, we can argue that those centers work as attractors primarily due to intense programmatic influence and less due to their strategic position in the conventional street network.

An example of such a business district is La Defense in Paris, as discussed by Ratti (2004). This district was willfully created outside of the historic city center.[24] Another example is 4th Levent in Istanbul which was first developed with residential projects in the 1950s and continued with high rise commercial and office buildings. Although the location of the 4th Levent district is relatively peripheral, the district attracts spatial activity more than space syntax analysis may suggest (Fig. 2).

Another case where movement distribution may not be sufficiently explained solely by street network analysis covers urban environments with green open spaces such as city parks. Recent park projects offer more fluid relationships with the urban fabric and flexible uses within green spaces.[25] This new design capacity makes those parks potentially strong attractors of movement, especially in overcrowded cities where people's interaction with natural settings are otherwise limited (Fig.3).

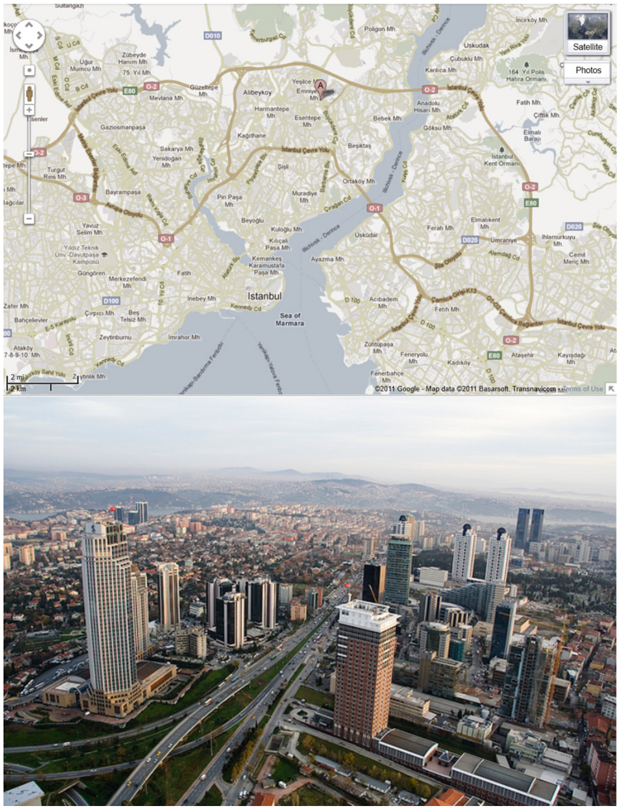

Figure 2. Map of Istanbul marking (A) location of 4th Levent district (top image); aerial view of 4th Levent district (bottom image, photo: Ferad Zyulkyarov

Figure 3. Central Park, New York, 2008 (photo: first author).

While attraction to parks with natural settings may depend on other factors including design and other programmatic activities offered, a number of findings in environmental cognition reinforce our identification of parks with natural settings as possible attractors. These findings suggest that natural settings impact mental activities, attitudes and actions, as these settings prompt fascination and restorative break from work related routines. [26,27] Parks with natural settings may be perceived as prominent elements and registered differently in our cognition when experiencing a city. Thus some streets and their leading routes may have a different effect due to their environmental content.[28] People also develop internal representations based on for example whether street segments have strong boundary conditions or not.[29, 30] This argument holds true when actual spatial experiences of people, which is based on understanding space gradually, is taken fundamental to modeling movement distribution, instead of theoretical and "top-down" reading of street network. [31] From this point of view, considering the green open spaces as attractors of spatial activity challenges the abstract nature of street network analysis where spatial entities are removed from their environmental contents. The capacity of green open spaces to impact movement motivates the need for a finer grain analysis where spaces can be read differently based on their spatial definition through natural or built elements.

There are other possible influencers beyond building densities and green open spaces. Declining economic activity in post-industrial cities may cause an opposite trend where street networks are no longer meaningful in predicting movement patterns where there is no programmatic content associated with space. The potential effects of green open spaces on movement, on the other hand, hold true particularly when environmental sources and interaction with nature appears to be a rare opportunity for residences in overcrowded cities. In cities, residential zones with natural settings gain greater economic value and thus can be the source of uplifting the conditions of high density urban environments. Exploring whether green open spaces attract people and create another layer of movement economy can lead to an understanding of the influence of urban greening on gentrification in cities. The urban conditions discussed here calls for a more detailed and informative modeling of potential spatial activity within city morphologies.

A NEW MODELING APPROACH DETECTING THE EFFECT S OF PROGRAMMATIC AND ENVIRONMENTAL CONTENT OF SPACE

In this section, we introduce a general analysis model to explain movement with factors other than (yet in addition to) street networks, such as strong programmatic and environmental content exemplified with two cases above. We propose an analysis approach based on the Markov chain probabilistic model that can weight spatial units based on their programmatic and environmental content along with their network properties. This approach uses the Markov chain paradigm to enable normalized weighing in probabilistic models of movement distribution.

Markov chains are a discrete-state representation of how behavior of entities (such as humans) transitions from one state to another over time.[32] Markov chains have been widely used in diverse fields to model state transition dynamics to generate predictive analyses of short- and long-term behaviors in both simple and complex systems. These previous applications include performing speech recognition, predicting optimum Internet search results and detecting behavior of computer infections spread by the Internet. .[33,34,35] There are abundant mathematical [36] and computational (www.mathlab.com) tools to analyze and generate predictions from Markov Chain models. The Markov chain formalism provides a promising approach to model dynamic space usage that incorporates normalized weightings of attractors' impact on path choice to predict long-term behavior.

The basis of the Markov chain approach is a probabilistic model of the transitions between units and entities. The model establishes the potential use of street segments, for example, within the probabilistic values expressing normalized weights given to each street segment or other spatial units. Figure 4 contains a representation of how a pedestrian might move between various spatial units in a (very) small city block. The labeled circles in Figure 4 represent the location states of the pedestrian and the direct arcs represent how the pedestrian could move from one state to another. In Figure 4 there are five spatial units: Building 1, Building 2, Street 1, Street 2 and Park 1. The directed arc from Building 1 to Street 1 in the model represents that it is physically possible to move from Building 1 to Street 1. (The arcs correspond to links in the mathematical graphs used in conventional space syntax analysis). According to the model in Figure 4, it is possible for a pedestrian to go from Building 1 to Park 1 by going from Building 1 to Street 2 and then from Street 2 to Park 1.

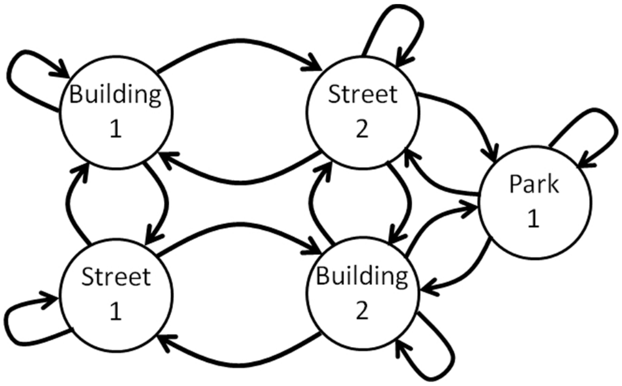

Figure 4. Logical State Location Transitions near a Park

By convention, every spatial unit has an arc that loops back to itself. This self-arc is to represent that pedestrians can also stay in their state location if they choose to do so. These self-arcs thus represent the possible hierarchical benefits of our Markov chain approach – each state location may be comprised of internal state locations. In other words, each state location may have attracting power and thus motivate movement to be concentrated in that location. A street may consist of multiple storefronts and a more detailed model could represent that it is possible for a pedestrian to move between and pause at storefronts along the street. We could improve our high-level state transition model in Figure 4 by replacing the street states with another set of state transitions to represent how a pedestrian could pause at individual stores.

We use the state transition model in Figure 4 to build the Markov chain model seen in Figure 5. A Markov chain captures the "probability" or "likelihood" of various state transitions occurring. For the Markov chain model in Figure 5 we label an arc from one spatial unit to another with the probability of a pedestrian moving from that first spatial unit to the other over a given time period. The example in Figure 5 represents that a pedestrian starting in Building 1 has a 70% probability of staying in the building, a 10% of moving to Street 1 and a 20% probability of moving to Street 2 over any given time period.

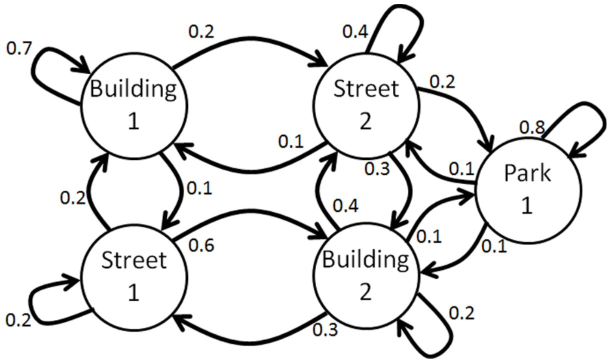

Figure 5. A Markov Chain Model of State Location Transitions near a Park

Once we have a Markov chain model of pedestrian' movements as exemplified in Figure 5, we can perform extensive analyses to estimate for example the likelihood that a pedestrian is in any given spatial unit at any given time, the likelihood that a pedestrian will eventually visit a spatial unit within some time period, or the likelihood of a pedestrian choosing a particular path between two spatial units using common mathematical[37] and computational (www.matlab.com) tools. We can also aggregate the pedestrian model to predict the likelihood of specific locations becoming crowded given that there are many pedestrians.

The major challenge and benefit of Markov chain modeling lie in the identifying the probability labeling of the arcs. There are a number of statistical and mathematical approaches to estimating these weightings based on experimental observation.[38] The weights in the Markov chain can be assigned based on sampled traffic analysis, among other methods. For sampled traffic analysis, we may sample the historical movement paths of pedestrians to estimate the state transition probabilities. For example, we may sample 100 visitors to Building 1 in a specific time period and observe that 70 stay in Building 1, 20 move to Street 2 and 10 move to Street 1. From this sampling we would assign the state transition probabilities of 0.7, 0.2 and 0.1 respectively of staying in Building 1, moving to Street 2 and moving to Street 1.

We can also assign weightings based on psychological models as well as space network models. For example, in a space network model we can use the angular integration and choice measures[39, 40] to assign weightings on street segments. Additionally, the Markov chain model allows weightings on the basis of other quantifiable values of attractors, such as building size and density, as well as attracting power of green open spaces. For example, while some street segments can have normalized weightings based on their network properties such as angular integration and choice measures, weightings can also be assigned for high rise commercial buildings, attractiveness of green open spaces or their any other programmatic and environmental content. One caveat is that to calculate the attracting power of green open spaces on the basis of environmental cognition studies accounting people's cognitive patterns. The attracting power of parks could be defined within quantitative measures based on further research on park size and other design and natural diversity variables that influence effect on people's behavior.

Markov chains could be validated using standard approaches currently used in other domains. Some of these standard approaches are further sampling of pedestrian movements in the modeled space and generating predictive movement probabilities distributions over multiple time frames in order to confirm that these patterns match observed reality of groups of pedestrians and the current theory.

CONCLUSIONS

Our discussion of the Markov chain model presents the first insights from an ongoing investigation into how to predict spatial activity distributions influenced by other than street networks. The space syntax framework has an established place within other urbanism models derived from space paradigm. Yet, the street network analysis of space syntax has some limitations stemming from the abstract nature of modeling potential spatial activity distributions merely on the basis of relational aspects. This model remains as an abstract representation against complex set of factors that may also influence distribution of spatial activity. More importantly, street segment modeling based on the fewest number of turns and least angular change make the most sense for cities such as London that have grown organically and show a certain degree of uniformity in term of building densities. The Markov chain model can be particularly useful to analyze movement distributions in cities that have had less organic development patterns, shaped with top-down planning decisions or eclectic zoning rules. Such development patterns may manipulate movement beyond the patterns generated by street networks. The environmental content of spaces, such as natural settings may work as an attractor of movement as these settings become increasingly valuable for spatial experience and social interaction patterns in overcrowded cities. Analyzing the movement distributions influenced by such complexities can aid exploring the implications of developments such as higher density in segregated locations, decaying central districts, and movement economy manipulated by green open spaces.

The Markov chain model differs from street network analysis in a number of ways. First, the Markov chain approach can account not only for streets but also for buildings and open spaces as spatial entities where people can pass through or loop back. Second, the Markov chain approach takes movement as a function of probabilistic model determined by network properties, programmatic and environmental content of street segments and other spatial units. Third, the Markov chain approach gives researchers the opportunity to manipulate the analysis model based on the relative effects of the spatial units due to their programmatic and environmental content. Utilizing the Markov chain formalism, our proposed analysis allows for normalized weightings of streets and other spatial units, based on network properties as well as programmatic and environmental attractors contained within those spatial units.

Motivated by these first insights, our discussion intends to improve upon previous spatial analysis approaches to be able to analyze complex and dynamic spatial conditions of cities. Further research is needed and is ongoing to elaborate this model to precisely determine the relative importance of attractors and other properties.

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